

Toothpick Patterns

Focus Problems: Stepping Stones Handout

Instructions

The illustration shows how a gardening company defines the sizes of its gardens, and shows how it puts stepping stones around the outside of the gardens.

- Organize the information from the first three designs into a table.
- For a garden of size 4, how tall and wide will the garden be? (Four tall and five wide. Be sure you understand why!)
- Extend your table to a garden of size 5.
- Look for a pattern to calculate the number of stones for each garden size. How would you calculate the number of stepping stones you need for a garden of size 10?
- What if your garden size is 100—how many stones do you need?
- Generalize* your calculation: make a formula for garden size n . What's your formula?
- Using a diagram, describe how the formula corresponds to the pattern of stepping stones.
- Find a different equation for the number of stones for a border of size n . Explain it.
- Show that the two formulas are *equivalent*.

size	pattern
1	
2	
3	

Sample Table

Garden Size	Tall	Wide	# stones
1	3	4	10
2	4	5	14
3	5	6	18
4	6	7	22
5	7	8	26
10	12	13	46
100	102	103	406

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Focus Problems: Stepping Stones Handout contd.

Answers, Approaches, and Solutions

Sides and Corners:

For garden n , there are n stones directly to the left and right. That's $2n$. Then there's one more than that on the top and bottom, so that's $2(n + 1)$. Then there are four corners, for 4 more. Total: $2n + 2(n + 1) + 4$.

Fenceposts and Rails:

The left and right sides, if you go all the way to the top and bottom, you have $n + 2$. That's $2(n + 2)$. Add the "rails" top and bottom and you get $2(n + 2) + 2(n + 1)$.

Recursive Approach:

The first one has 10, and each one after adds four more stones. So $S(1) = 10$, and $S(n + 1) = S(n) + 4$. (Using S for "stones") This is correct, but hard to use for size 100!

Probe to see how a student using this approach actually gets the answer. Usually they use some other way they just can't express as a formula—until you help them see how. One possibility is to say you start with 10, and then each step after 1 adds 4, so the formula is $S(n) = 10 + 4(n - 1)$.

Simplified formula: Any of these simplify to $S = 4n + 6$.